

# Superfields on a Graph



グラフ理論でいうところの「グラフ」の上の場の理論  
= Graph Field Theory を考えます。  
グラフの頂点や辺の上に超場(superfield)を乗せて  
みましょう。

Moose とか Quiver とか  
昔からありまして . . .

空間の離散化 . . .

Dimensional Deconstruction . . .

Higgsless theory . . .

D-brane と Quiver , M-theory, Matrix theory と . . .

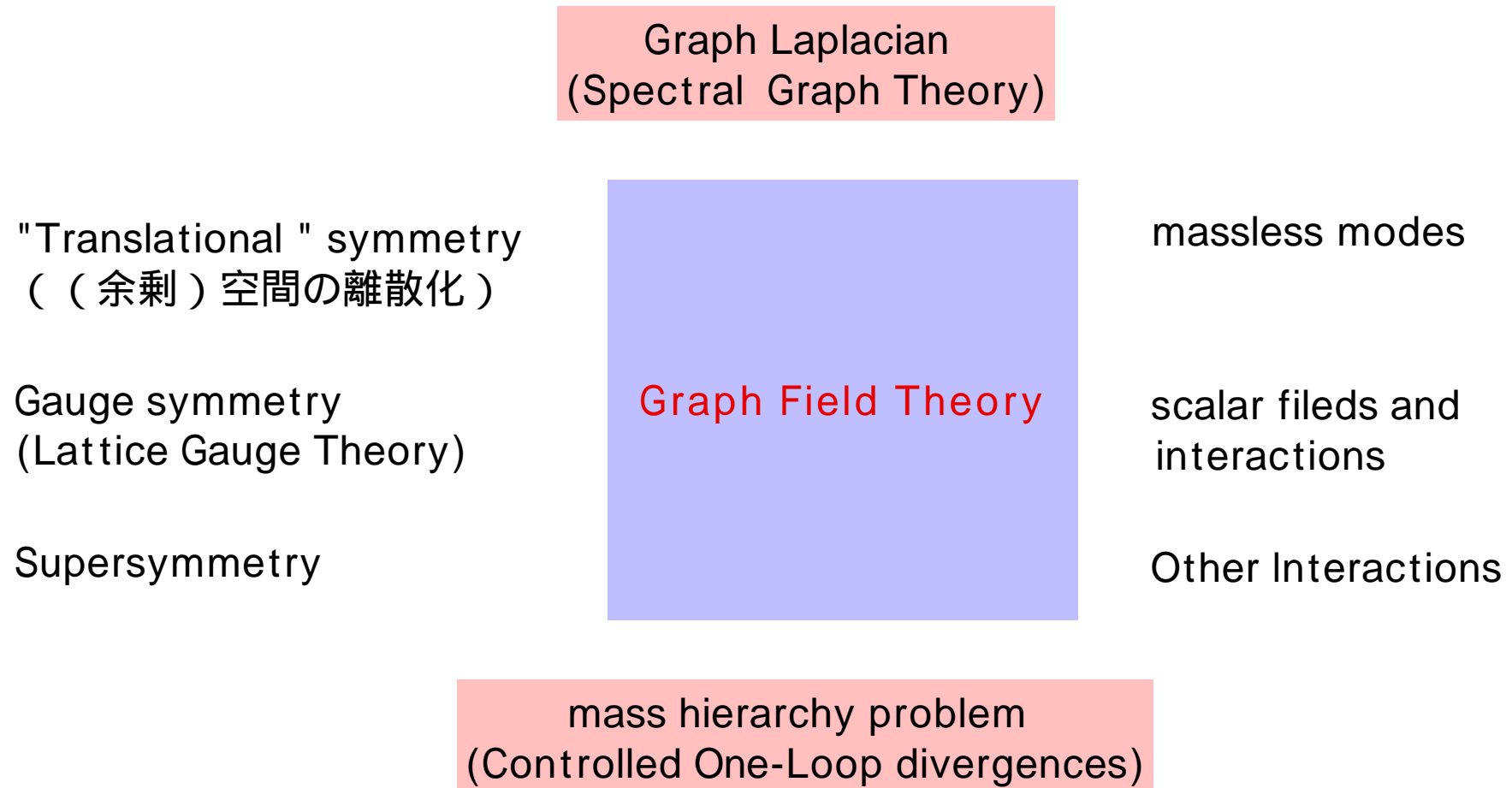
わたくし, よくわかってないことが多いですが,

みんな **グラフ** と関連

高次元超対称理論の四次元超場による記述 . . .

これもグラフに乗っかるといいのではないのでしょうか

4次元と高次元のはざまの理論



## Graph Field Theory の超対称化

1. どうせ高エネルギーでは超対称な何かなのでしょう？
2. いろいろな場のラグランジアンを定める  
もっともらしいさぐり方。

# Non-interacting Wess-Zumino model

$$y = x + i \quad \text{---}$$

chiral superfield  $\varphi_v = \varphi_v(y) + \sqrt{2} \theta \psi_v(y) + F_v(y)$

をグラフの頂点に

chiral superfield  $\varphi_e^c = \varphi_e^c(y) + \sqrt{2} \theta \psi_e^c(y) + F_e^c(y)$

をグラフの辺に対応させる。

$$L_{WZ} = \sum_v \left[ \varphi_v \left( \frac{d}{dt} \varphi_v - \psi_v \right) \right] + \sum_e \left[ \varphi_e^c \left( \frac{d}{dt} \varphi_e^c - \psi_e^c \right) \right] + m \sum_e \left[ \varphi_e^c \left( \psi_{t(e)} - \psi_{o(e)} \right) \right] + \text{h.c.}$$

## Global symmetry

$$\begin{array}{ccc} & e^{-2} & \\ v & & v \\ & c & c^2 \\ c & & e \\ e & & e \end{array}$$

$$\begin{aligned}
& L_{\nu V} \left[ - \frac{\mu}{\nu} \frac{t}{\nu} - i \frac{\mu}{\nu} + \left| F_{\nu} \right|^2 \right] \\
& + L_{e E} \left[ - \frac{\mu^c}{e} \frac{t^c}{e} - i \frac{\mu^c}{e} + \left| F_e^c \right|^2 \right] \\
& + m_e E \left[ - \frac{c}{e} (E^T F)_e - F_e^c (E^T)_e + \frac{c}{e} (E^T)_e + \text{h.c.} \right]
\end{aligned}$$



補助場を消去すると

$$\begin{aligned}
 & \nu V \left[ \begin{array}{c} - \mu \quad \dagger \quad \mu \\ \mu \quad \nu \quad \nu \end{array} - m^2 \begin{array}{c} \dagger \\ \nu \end{array} (E E^T) \begin{array}{c} \\ \nu \end{array} \right] \\
 & + e E \left[ \begin{array}{c} c \dagger \quad \mu \quad c \\ \mu \quad e \quad e \end{array} - m^2 \begin{array}{c} c \dagger \\ e \end{array} (E^T E) \begin{array}{c} c \\ e \end{array} \right] \\
 & + \nu V \left[ \begin{array}{c} - i \quad \mu \\ \nu \quad \mu \end{array} \overline{\nu} \right] + e E \left[ \begin{array}{c} - i \quad \mu \quad c \\ e \quad e \quad \mu \end{array} \overline{e} \right] \\
 & + m e E \left[ \begin{array}{c} c \\ e \end{array} (E^T) \begin{array}{c} \\ e \end{array} + \overline{c} \begin{array}{c} (E^T) \\ e \end{array} \right]
 \end{aligned}$$

# Abelian vector field

$$V_v = - \int_{\mu} A_{\nu}^{\mu} + i \int_{\nu} - i \int_{\nu} + \frac{1}{2} \int D_{\nu}$$

を各頂点におく。

一方，辺上にchiral superfieldを設定。

$$S_e = \frac{e + ia}{2} - i \int_{\mu} + i \int_{\mu} \frac{e + ia}{2} + F_{Se} + \frac{1}{2} \int_{\mu} + \frac{1}{4} \int_{\mu} \frac{e + ia}{2}$$

## 頂点をつなぐ辺・・・Stueckelberg term

$$\begin{aligned}
 L_s = & \int_e E \left[ \frac{1}{2} (V_{t(e)} - V_{o(e)} - S_e - \bar{S}_e)^2 \right. \\
 & \left. - \frac{1}{2} (E^T A^\mu + a^\mu)^2 + (E^T)^\mu + \bar{(E^T)}^\mu \right] \\
 + & \int_e E \left[ 2 |F_{Se}|^2 - i \mu \bar{\mu} - \frac{1}{2} (\mu^\mu)^2 + (E^T D)^\mu \right]
 \end{aligned}$$

## 頂点上のkinetic term

$$L_v = \frac{1}{16g^2} (WW | + \overline{WW} | \_ ) , W = -\frac{1}{4} \overline{D} D e^{-2V} D e^{2V}$$

といっしょに,  $L = L_v + v^2 L_s$  とし,  $S_e = S_e / v$ ,  $V_v = gV_v$  とrescale

$$\begin{aligned}
L = & -\frac{1}{4} F_{\nu}^{\mu} F_{\mu\nu} - \frac{1}{2} g^2 v^2 (E^T A^{\mu} + \frac{1}{g v} a)^2 \\
& + \frac{1}{v} V \left[ -i \frac{1}{v} \frac{\partial}{\partial x^{\mu}} \right] + \frac{1}{e} E \left[ -i \frac{1}{e} \frac{\partial}{\partial x^{\mu}} \right] \\
& + g v \frac{1}{e} E \left[ \frac{1}{e} (E^T) + \frac{1}{e} (E^T) \right] \\
& + \frac{1}{e} E \left[ -\frac{1}{2} \left( \frac{1}{e} \right)^2 - \frac{1}{2} \frac{1}{e} (E^T E) + 2 \left| F_{Se} \right|^2 \right] \\
& + \frac{1}{v} V \left[ \frac{1}{2} (D + g v E)^2 \right]
\end{aligned}$$

# Abelian gauge coupling to matter

Local symmetry (グラフの各頂点)

$$\begin{aligned}
 & \bar{\psi}_v e^{-2igV} \psi_v + \bar{\psi}_e e^{2igV} \psi_e \\
 L_{WZ} = & \int d^4x \left[ \bar{\psi}_v e^{2igV} \psi_v + \bar{\psi}_e e^{-2igV} \psi_e \right. \\
 & \left. + m \int d^4x \left[ \bar{\psi}_e \left( e^{2igS_e/v} \psi_e - \psi_e \right) + \text{h.c.} \right] \right]
 \end{aligned}$$

$$\begin{aligned}
& L_{\nu V} \left[ -\frac{1}{2} \bar{\nu} \not{\partial} \nu - i \bar{\nu} \not{D} \nu - i\sqrt{2}g(\bar{\nu} \not{D} \nu - \bar{\nu} \not{\partial} \nu) \right] \\
& + \frac{1}{2} \left[ |F_{\nu}|^2 + gD_{\nu} |\nu|^2 \right] \\
& + L_{e E} \left[ -\frac{1}{2} \bar{e} \not{\partial} e - i \bar{e} \not{D} e + i\sqrt{2}g(\bar{e} \not{D} e - \bar{e} \not{\partial} e) \right] \\
& + \frac{1}{2} \left[ |F_e^c|^2 - gD_{o(e)} |e^c|^2 \right] \\
& + m_{e E} \left[ -\bar{e}^c (\hat{E}^\dagger F_e) - F_e^c (\hat{E}^\dagger e) + \bar{e}^c (\hat{E}^\dagger e) + \text{h.c.} \right] \\
& + \frac{m}{v} \left\{ e^{(i a_\nu + i a_e)/v} \left[ i\sqrt{2}(\bar{e}^c e_{t(e)} + \bar{e}^c e_{t(e)}) + \bar{e}^c_{t(e)} (2F_{Se} + e_{t(e)}/v) \right] + \text{h.c.} \right\}
\end{aligned}$$

$$D_{\nu}^{\mu} = \left( \begin{array}{c} \mu \\ \nu \end{array} + ig A_{\nu}^{\mu} \right)$$

$$D_{\nu}^{\mu} = \left( \begin{array}{c} \mu \\ \nu \end{array} + ig A_{\nu}^{\mu} \right)$$

$$D_{e}^{\mu c} = \left( \begin{array}{c} \mu \\ o(e) \end{array} - ig A_{o(e)}^{\mu} \right)$$

$$D_{e}^{\mu c} = \left( \begin{array}{c} \mu \\ o(e) \end{array} - ig A_{o(e)}^{\mu} \right)$$

$$(\hat{E})_{ve} = \left\{ \begin{array}{ll} 1 & \text{if } v=o(e) \\ -e^{(-ia)_e/v} & \text{if } v=t(e) \\ 0 & \text{otherwise} \end{array} \right.$$



# D-term (Fayet-Iliopoulos) breaking

$$L_{FI} = \frac{1}{2} D^2$$

SUSY breaking

modifying mass spectra (no massless modes)

# non-Abelian vector field

よく知られたゲージ変換

$$e^{2V_v} \quad e^{2\bar{V}_v} \quad e^{2V_v} \quad e^{2\bar{V}_v}$$

と, "adjoint" 的 chiral superfield

$$\begin{aligned}
 & e^{-2t(e)} \quad e^{2\alpha(e)} \\
 & + \frac{1}{4} \left( \frac{1}{2} U_e - 2i \right) + \frac{1}{4} \left( \frac{1}{2} U_e + i \right) + \frac{1}{4} \left( \frac{1}{2} U_e + F_e \right) \\
 & + \frac{1}{4} \left( \frac{1}{2} U_e - 2i \right) + \frac{1}{4} \left( \frac{1}{2} U_e + i \right) + \frac{1}{4} \left( \frac{1}{2} U_e + F_e \right)
 \end{aligned}$$

$$e E \operatorname{Tr} \left( \overline{U}_e e^{2V_{t(e)}} U_e e^{-2V_{o(e)}} \right)$$

$$e E \operatorname{Tr} \left[ - (D_\mu U_e)^\dagger (D^\mu U_e) + U_e^\dagger D_{t(e)} U_e - U_e D_{o(e)} U_e^\dagger + |F_e|^2 - 2i \overline{U}_e \overline{D}_\mu U_e + 2 U_e (U_{t(e)}^\dagger - U_{o(e)}^\dagger) + 2 U_e (\overline{U}_{t(e)} - \overline{U}_{o(e)}) \right]$$

$$D^\mu U_e = \partial^\mu U_e + i A_{t(e)}^\mu U_e - i U_e A_{o(e)}^\mu$$

$$\overline{D}^\mu U_e = \partial^\mu \overline{U}_e + i \overline{U}_e A_{t(e)}^\mu - i A_{o(e)}^\mu \overline{U}_e$$

"fundamental"

$$v \quad e^{-2} \quad v \quad c \quad c \quad e^{2} \quad t(e)$$

$$c \quad ( \quad - \quad ) + \text{h.c.}$$

"adjoint"

$$v \quad e^{-2} \quad v \quad e^{2} \quad v \quad c \quad e^{-2} \quad \alpha(e) \quad c \quad e^{2} \quad t(e)$$

$$\text{Tr} \quad c \quad ( \quad - \quad ) + \text{h.c.}$$

## まとめ

地道に作りました。

## これから

non-linear sigma model  
(constraint or  $\log(\dots)$ )

symmetry breaking, including quantum effects

使えるモデル 種々の群